Water Rates from Piecewise Functions 03-19

Note : this pdf note was generated directly from a *Mathematica* notebook. [rob r, 2008-03-15, 19]

This exercise was occasioned by one of my colleagues (Tom Gaskill) describing some water usage data and ideas about its' analysis. It looked like the water usage was billed by a procedure that had stepwise increments as water usage increased, This is just like price breaks in purchasing. For example, the first 100 items are a certain price while those over 100 are priced at a reduced rate while those over 1000 are priced even lower. The function could go the other direction as well though, as a commodity or service became more scarce/valuable in larger volumes (like space on a life boat or luggage weight on an airline!). So, I was playing with some *Mathematica* functions to try to express this. *See the provider[. .] function below as well asplots of cost curves associated with its use.*

Note : There is an alternative way to show these piecewise functions. An example is at the end of this tutorial.

Once I have the provider function, I will show a few plots of the cost curve for some made-up data.
For example, for provider1, say the '*fixed'* cost is 2 units, which is the cost before the tap is even opened.
I use small values, but of course, these can be scaled. Then, suppose the price *p1* for the first 5000 gallons is 7 units. In the *provider* function, I also pro-rated the cost to be: 7 * gallons of usage/5000 in case someone used less than 5000.
So the cost for using all 5000 gallons would be 2 + 7 = 9. If less than 5000, I pro-rated it as above.
Then, suppose the next block of water is priced at say, *p2* = 3 units for a block, say
between 5000 and 10000. For this block of water the cost would be
fixed + p1 + p2 * (gallons - 5000)/ (10000 - 5000), So if someone used 8000, then I would calc this as :
fixed + p1 + p2 * 3/8

fixed = $ fixed cost of services from the provider

gal = cumulative gallons of usage (It's best to use low numbers here and scale)

p1, p2, p3, . . .price/block specific to blocks 1, 2 3, . . .
where a 'block' means a range of water allocation like
between 5000 and 10000 gallons
If the price for that block is $10, then the cost is $10/5000 gallons
or $1/500 per gallon within that range

The plan here is to represent the providers rates per
cumulative gallons used , specific to each block of water allocation.

** ** ** ** ** ** ** ** ** **

Basic Provider Piecewise Function

This function takes an identifier of the water provider,
id = provider identifier
gal =gallons used, the variable whose value determines the cost.
fixed = fixed cost before any water is actually delivered
blocks - a list of 3 component lists, where each component is
{ p, lo, hi } where p is the block cost, lo is the extreme lower value
of the water block while hi is the high extreme value , e.g.
blocks = { { 7, 0, 5000}, {4, 5000, 10000}, { 1, 10000, 18000} }
For example, block 2 has a price of 4, and the range is between 5000 and 10000.
Just as an aside, it's best to use small numbers and scale later

```
Clear[provider, a, b, c, s, t, u, v, b1, b2, temp, arg, numberBlocks, blocks, p, lo, hi, i]

provider[id_, gal_, fixed_, blocks__] :=
 Module[{price, temp, arg, numberBlocks, p, lo, hi},
  numberBlocks = Length[blocks];
  Print["numberBlocks  ", numberBlocks];
  price = fixed;
  arg = {}; (* start the  build vector of args for Piecewise *)
  For[i = 1, i ≤ numberBlocks, i++,
   p  = blocks[[i, 1]]; (* block price *)
    lo = blocks[[i, 2]]; (* low range value of block  water usage*)
   hi  = blocks[[i, 3]]; (* high range value of block water usage *)
   temp = {price + p * (gal - lo) / (hi - lo),
     lo < gal < hi}; (* low to high interval *)
   arg = Append[arg, temp]; (*build up Piecewise arguments *)
   (*Print["i ", i, " price " ,price, "  ",arg]; *)
   price = price + p; (* set up for next iteration *)
  ]; (* end For loop *)
  Piecewise[arg] (*load arguments to piecewise function *)
 ]
```

- **** ** ** ** ** ** ** ** ** *** End Function ********** ** ** ** ** ** ***

Provider 1 Data

- **Note: **** This is strictly hypothetical data made up to play with**

Just to start, I am using just three blocks of water rates, (the program will handle any number of blocks).
fixed price = 3
block 1 :
price = 3
low range = 0, high range = 5 (maybe scale by 1000?
block 2:
price = 2
low = 5, high = 10
block 3:
price = 1.5
low = 10 high = 25
Note that the blocks are lists and are enclosed in a list. That way, I can tell how many blocks there
are... The function displays its interior structure as a piecewise function. So, for gallon usage between 0 and 5, I would bill the customer at $3 fixed charge, plus prorated cost from 0-5 with a
cost of $3 * (gallons - 5000)/ (5000 -0). A total of $6 for 5000 units of water usage.

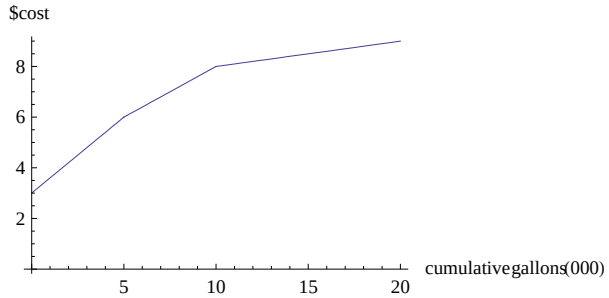
```
provider1 = provider["Joe", gal, 3,
  {{3, 0, 5}, {2, 5, 10}, {1.5, 10, 25}}]
```

```
numberBlocks  3
```

$$\begin{cases} 3 + \frac{3\,gal}{5} & 0 < gal < 5 \\ 6 + \frac{2}{5}\,(-5 + gal) & 5 < gal < 10 \\ 8 + 0.1\,(-10 + gal) & 10 < gal < 25 \end{cases}$$

```
p1 = Plot[provider1, {gal, 0, 20},
  PlotLabel → "Provider 1 Cost  per gallons of  usage (pro-rated)", AxesOrigin → {0, 0},
   AxesLabel → {"cumulative gallons(000)", "$cost "}]
```

Provider 1 Cost per gallons of usage (pro−rated)



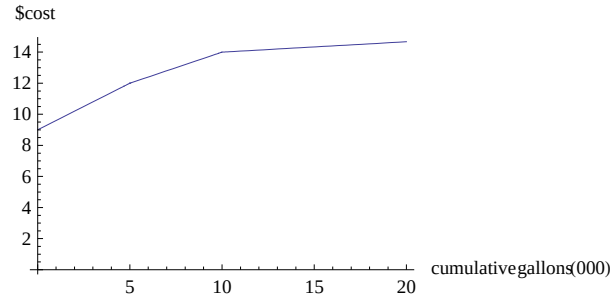
```
provider2 = provider["Sammi", gal, 9,
  {{3, 0, 5}, {2, 5, 10}, {1.0, 10, 25}}]
```

```
numberBlocks  3
```

$$\begin{cases} 9 + \frac{3\,gal}{5} & 0 < gal < 5 \\ 12 + \frac{2}{5}\,(-5 + gal) & 5 < gal < 10 \\ 14 + 0.0666667\,(-10 + gal) & 10 < gal < 25 \end{cases}$$

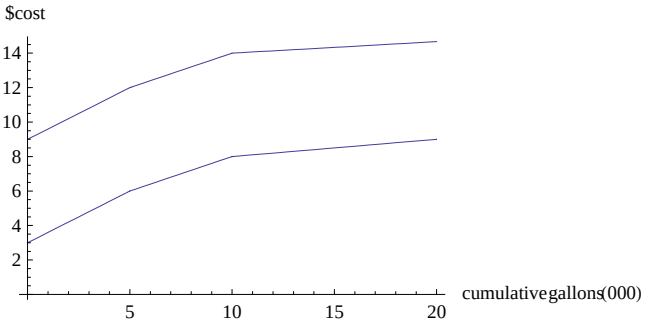
```
p2 = Plot[provider2, {gal, 0, 20},
  PlotLabel → "Provider 2 Cost  per gallons of  usage (pro-rated)", AxesOrigin → {0, 0},
  AxesLabel → {"cumulative gallons(000)", "$cost "}]
```

Provider 2 Cost per gallons of usage (pro−rated)




```
Show[p1, p2, PlotRange → Automatic, PlotLabel → "Provider 1 & 2"]
```

Provider 1 & 2



■ An Alternative Piecewise Description

Mathematica has built in function called Piecewise, that I used above, that will let me enter functional values on a per interval basis. The approach below is more verbose than the *Piecewise* function, but will allow a bit more variation.

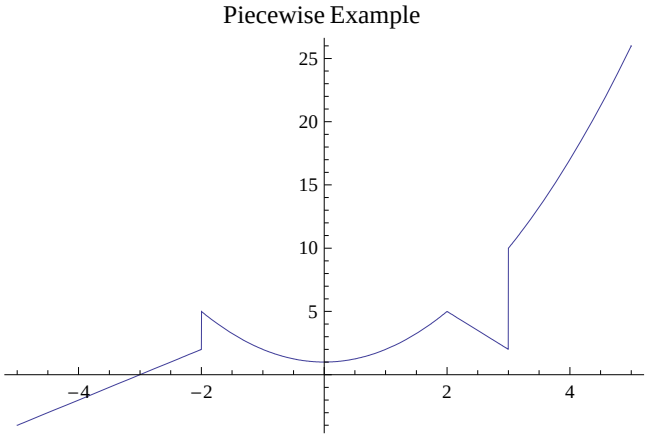
```
Clear[h, x]
```

```
h[x_] := x^2 + 1 /; -2 < x ≤ 2 || x > 3 (* define h over disjoint intervals*)
```

```
h[x_] := 11 - 3 x /; 2 < x < 3
```

```
h[x_] := 6 + 2 x /; x ≤ -2
```

```
Plot[h[x], {x, -5, 5}, PlotLabel → "Piecewise Example ",
 AxesOrigin → {0, 0} ]
```



Summary

The application here is just a toy example but the math is not trivial. Further work could include various tables generated for different scenarios and additional function can be fairly easily defined. *Mathematica* is extensible in limitless way, if I just knew enough!