Trigonometry Basics
[***Draft 2009-08-14]

Hello all, this revision includes some numeric examples as well as more trigonometry theory. This set of notes is intended to accompany other tutorials in this series. I have developed the *Law of Cosines* from basic ideas and then go on to show how the *Dot Product* falls out of this derivation. I also put in the *Cross Product* as an expression of the area of a 2-dimensional parallelogram as another example of the fusion of trig and geometry. For students looking into physics, engineering, or social statistics, knowing about both the Cross product and the Dot Product will be helpful. Other tutorials on this site, such as *Introduction to EDA*, *Fivenumber BoxPlots*, *Business Trend Analysis,* are based on the discussions in this tutorial. There is another tutorial, *Vector Arithmetic and Vector Operations* that gives more detail about vectors and their Operations.In that tutorial I talk about vector and vector spaces as well as the uses of the Dot and Cross Products.

As a bonus for those who want a little extra, I have put in an abbreviated application of Euler's formula using complex numbers, to derive several trig angle identities. This is definitely supplemental material but, I couldn't resist pointing out how *all* trig identities can be derived from this formula plus just a little knowledge about complex variables!

Conventions and Notation:

I will use the following symbols and notations:

1. t - denotes an angle (in degrees). I also use 'theta' occasionally
2. s - denotes an angle (in degrees)
3. a, b, c, d, e, f, g, h, k, m - denote lengths, regular real numbers.
4. **A, B, X, Y**- bold symbols denote vectors, that is, directed line segments generally, but we will specifically use these to represent sample or population observation vectors or statistical parameters
   or physical quantities that can be treated as vectors such as Force, Velocity, Acceleration, or Torque.
5. |A|, |B|, |X|, |Y| - denotes the length of these vectors.
6. {a1,a2}, {b1,b2}, {x1,x2}, {y1,y2} - denote coordinates, and will be used to show positions in 2-dimensional space. These are the x, y coordinates of the end point of a vector in 2-D space, or a general point in 2-D space.
7. {a1,a2,a3}, {b1,b2, b3}, {x1,x2,x3}, {y1,y2, y3}- will be used to show positions in 3-dimensional space. These are the x, y, z coordinates of the end point of a vector in 3-D space, or a general point in 3-D space.
8. To indicate a variable raised to a power, I will use the symbol" ^". So, if I want to show the square of the variable 'a', I will write a^2. If 'a' is to be cubed, I write a^3.
9. To indicate a square root I will use the notation Sqrt[x]. For example, Sqrt[16] = 4. I could also write this as 16^(1/2) or 16^0.5
10. To indicate (scalar) multiplication I will use a "*". So 4*3 = 12.

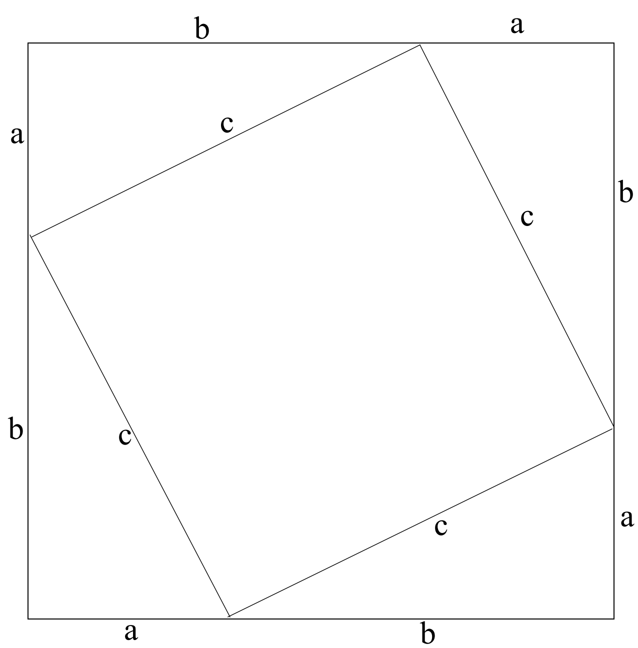
11. For vector multiplication I will use the Dot product, indicated by a dot, '•'. For example, the dot product of the two vectors A= {2,4} and B= {-3, 12} is written as {2,4} • {-3, 12} and results in the number (2* -3) + (4 * 12) = 42. As you can see, each component of the first vector is multiplied by the corresponding component of the second vector and these products are added up.
12. Another type of vector multiplication is the Cross product of two vectors, indicated by an 'x'. For example, the cross product of the two vectors A={2,4} and B{-3,12}, is written as A x B and the result is the vector perpendicular to the plane formed by A and B and of magnitude |A| * |B| * Sin[t], where 't' is the angle between them. This can be also be done using their components as well and works out to be:
    ((2*12 + (4*3)) **n** = 48 **n**, where **n** is a unit vector perpendicular to A and B giving the resultant direction of the Cross product.

First Things First - Pythogoras and His Right Triangle

I want you to be able to state and prove the Pythagorean theorem which says that for a right triangle, the sum of the squares of the sides adjacent to the right angle, equals the square of the opposite side (hypotenuse). Consider the diagram below (Figure 1 on page 3): I would like to show one way that demonstrates the conclusion of the theorem.

First draw a square where each side is made up of an 'a' length plus a 'b' length, arranged as shown in the diagram. This means that each side is 'a+b' units long. Then look at the diagram and consider the steps:

1. total area of the square, from an outside perspective, is (a+b)^2= (a+b) * (a+b) = a^2 + b^2 + 2 * a*b
2. now draw the four interior triangles as shown, and the interior square that has 'c' on each side. (notice that I don't know what 'c' is, at this point, but I know each of the long sides is the same, so call them 'c'.
3. each interior triangle has area of (a * b)/2 and there are four of these, so, if you add them up, that gives an area of 2 * a* b. (remember, a rectangle has area of length times width, or if the length were 'a' and the width 'b', then the area would be a*b. Now, each of the triangles shown is half the area of that rectangle.)
4. the interior square, with 'c' units on a side, has area of c*c, or c^2
5. total area of the square from an inside perspective is c^2 + 2*a*b
6. Since it's the same square, the outside and inside perspectives must match, so:
7. c^2 + 2*a*b = a^2 + b^2 + 2*a*b
8. The 2*a*b terms 'cancel' and the conclusion follows:
9. c^2 = a^2 + b^2



Outside perspective, area of square is:
(a + b)^2 = a^2 + b^2 + 2 a * b

Inside perspective, area of square is:

4 (a* b)/2 + c^2= 2*a*b + c^2

Equating the two perspectives:

2*a*b + c^2 + = a^2 + b^2 + 2*a*b

Therefore:
c^2 = a^2 + b^2

FIGURE 1. Pythagoras Theorem: Demonstration

Example of Pythagoras Theorem

Just to get this to sink in, here is a numeric example of the theorem.

let a = 3, b = 4, then what is c?

The outer square would be (3+4) * (3+ 4) = 9 + 2* 3*4 + 16 = 49

Each a-b triangle on the inside has area (3 * 4)/2 = 6

There are four of these for a total area of 4* 6 = 24

The interior square of side ‘c’ has area c* c = c^2 (which we don’t know yet)

So,

(3+4)(3+4) = 4 * (3*4)/2 + c^2 = 2* 3 * 4 + c^2

9 + 2* 3 * 4 + 16 = 2*3*4 + c^2

9+ 16 = c^2

25 = c^2

So, c^2 = a^2 +b^2 = 3^2 + 4^2

c= 5 (this is a well know example of a triangle, called a ‘3-4-5’ triangle, go figure! You can see how these triangles work by drawing a few. The most common is the 3-4-5 triangle as we just looked at. Other triangles that work out to whole numbers are: 5-12-13 and 7-24-25.

Trig Talk

The next section introduces the basic trig functions that you will need for the remainder of your life. You will be able to use this diagram (Figure 2 on page 5), to predict and inter-relate the behavior of the various trig functions, geometrically. This diagram will allow you to see the trig functions in terms of physical lengths of sides of *right triangles*.

I advise you to carefully study the diagram below and commit the first three functions, *Sin*, *Cos*, and *Tan*, to memory. After studying the diagram, see if you can trace out the consequences of changes to the angle 't'. For example, when 't' approaches zero, what happens to each of the functions such as Sin[t] or Tan[t]? You can tell what happens by imagining how the lengths of the various lines change. What happens when t approaches 90 degrees? For example, when t approaches zero, Sin[t] approaches zero (that is, length 'b' goes to zero), while Cos[t] approaches '1' (that is, line 'a' goes to '1'). However, the function 1/Sin[t] will blow up to larger and larger values as 't' goes to zero. This function, 1/Sin[t], is called Cosecant of t, abbreviated as Csc[t] and will be described in the next diagram. Similarly, the Tan[t] will go to zero as t does, but Cot[t] will blow up. Check out the other functions for their behavior and see if you can mentally trace out the consequences of the functions as t increases or decreases by looking at how the various lengths a, b, c, d, e, g behave. Note that all of these lengths are the sides of *right triangles*.

Some Common Angles, Their Trig Functions and Length Equivalences

(Ignore the last three columns for now, I will describe those in the next sections). Look at row one of the table and you will see that Cos[0 degrees] = 1. This is also represented by the length of side **a**, which extends to the edge of the circle at {1,0}, when the angle 't' is zero. This makes the length of **a** =1. Similarly, Sin[0] is 0 and is represented by the length of **b**, which you can see will go to zero as 't' approaches zero. Sin[45] and Cos[45] are special values since the are equal. In this case, **a** and **b** both have length of Sqrt[2]/2 = 0.7071. To check out these values, just draw those triangles on graph paper with the angles given by a protractor. Make sure that you pick units so that the hypotenuse is 'unit' length. That is, if you draw your triangle at 30 degrees and the hypotenuse is 4 inches, then that is the unit of length. If the opposite side is 2 inches, you have just verified that Sin[30] = 2/4 = 1/2.

As another check on these last two values for Sin and Cos of 45 degrees, consider that because of symmetry, length **a** is equal to length **b.** This happens since 45 degrees is halfway between 0 and 90. Since c = 1, then:

c^2 = 1 = a^2 + b^2 = 2 * a^2 (since a=b)

So a^2 = 1/2 and a = 1/Sqrt[2] = Sqrt[2]/2

ANGLE 't' (degrees)	Sin[t] == **b**	Cos[t] == **a**	Tan[t] == **d**	Csc[t] == **g**	Sec[t] == **e**	Cot[t] == **f**
0	0	1	0	VERY BIG	1	VERY BIG
30	1/2	Sqrt[3]/2	1/Sqrt[3]	2	2/Sqrt[3]	Sqrt[3]/1
45	Sqrt[2]/2	Sqrt[2]/2	1	2/Sqrt[2]	2/Sqrt[2]	1
60	Sqrt[3]/2	1/2	Sqrt[3]	2/Sqrt[3]	2	1/Sqrt[3]
90	1	0	VERY BIG	1	VERY BIG	VERY BIG

The Big Three Trig Functions: Sin, Cos, Tan

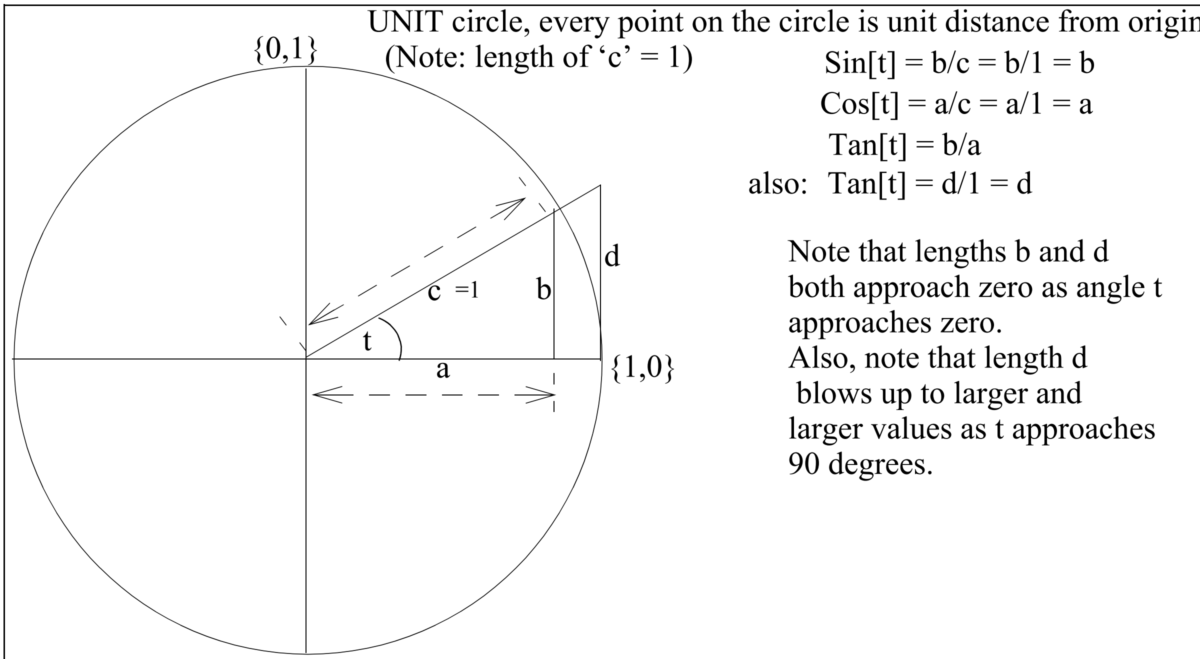


FIGURE 2. The Big Three Trig Functions, Sine, Cosine, and Tangent

Complements of Angles and Their Related Trig Functions

The Sin of an angle and the Cosine of an angle are intimately related as you can see from the diagram above. They are also related in a simple way when you rotate a right triangle as we will see from the diagram below. A rotation of a reference triangle by 90 degrees lets you talk about angles and their *complements*. Look at the triangle on the right in the diagram below and you will see Cos[s] = 3/5. Now consider rotating that triangle by 90 degrees, counterclockwise. That will give you the triangle on the left. See how the legs of the triangle switch? Now consider Cos[s + 90] and see how that ends up as asking for the Cos[t] = -4/5. Notice that the cosine is negative in this 2nd quadrant. Looking at the rotated triangle you can see that Cos[s+90] = -Sin[s] = -4/5

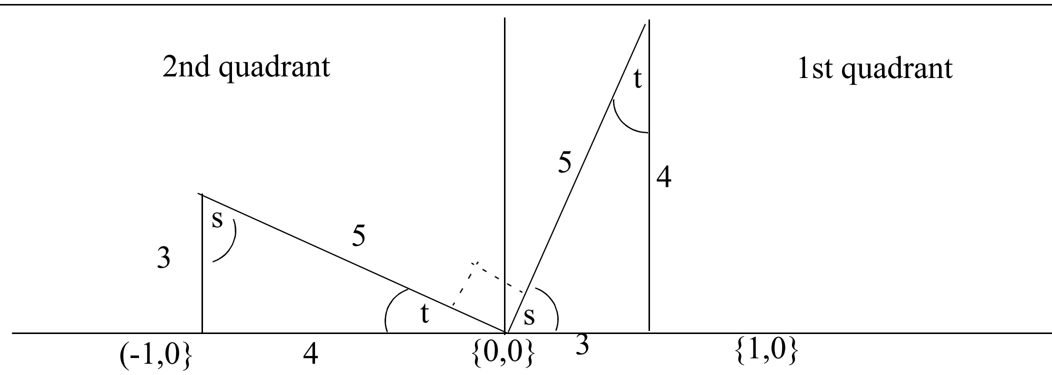


FIGURE 3. Complementary Sin and Cos: Cos[s + 90] = Cos[t] = -4/5 = -Sin[s]

These kinds of diagrams showing angles and their complements let you discover all of the trig complements. For example, (180 -s) is the angle t. Go ahead and see what other connections you can establish given that knowledge!

The Second Three Trig Functions: Csc, Sec, Cot

This is the same drawing as above, but with the other three trig functions included, together with their representative lengths. Again, the plan is to express trig functions in terms of simple lengths.

Before showing these three additional functions, let me step back and give some background for these functions.

A *secant* line of a curve is a line that intersects two or more points on the curve. The word secant comes from the Latin secare, for to cut. You can use the secant line to approximate the tangent to a curve, at some point P by running the secant line through successively closer points on the curve to P. The limit of these lines is the direction of a tangent line to the curve at the point P.

A *chord* is a segment of a secant line where both ends lie on the curve.

These next three functions are the reciprocals of the big three.

Cosecant[t] = Csc[t] = 1/Sin[t]

Secant[t] = 1/Cos[t]

Cotangent[t] = Cot[t] = 1/Tan[t]

This table presents a few angles, 't', and their trig functions: Note that Sqrt[x] means the square root of 'x'.

Some Common Angles, Their Trig Functions and Length Equivalences

Now you can see the complete equivalences between the lengths of the sides and trig functional values. *For these calculations I have set side 'c' = 1*. For example, the Sin[0 degrees] = 0 and is equivalent to the length of side **b**. The Csc[0] is the reciprocal of the Sin[0] and blows up. You can also see this from the fact that side **g** gets arbitrarily long.

ANGLE 't' (degrees)	Sin[t] == **b**	Cos[t] == **a**	Tan[t] == **d**	Csc[t] == **g**	Sec[t] == **e**	Cot[t] == **f**
0	0	1	0	VERY BIG	1	VERY BIG
30	1/2	Sqrt[3]/2	1/Sqrt[3]	2	2/Sqrt[3]	Sqrt[3]/1
45	Sqrt[2]/2	Sqrt[2]/2	1	2/Sqrt[2]	2/Sqrt[2]	1
60	Sqrt[3]/2	1/2	Sqrt[3]	2/Sqrt[3]	2	1/Sqrt[3]
90	1	0	VERY BIG	1	VERY BIG	VERY BIG

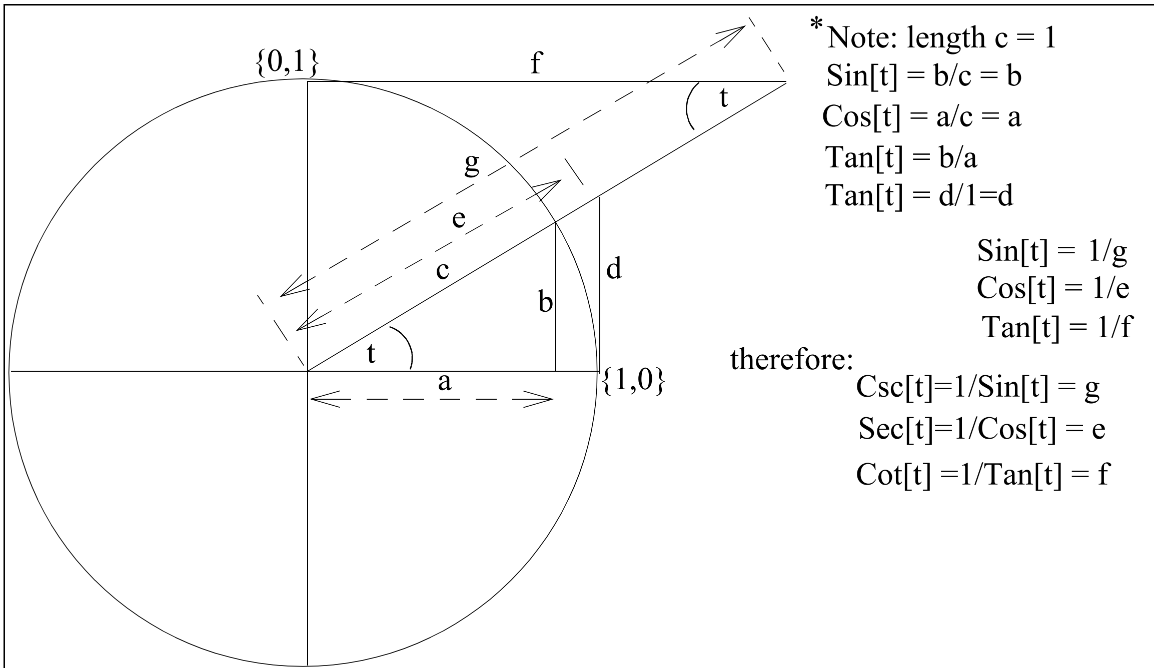


FIGURE 4. The Second Three Trig Functions: Cosecant, Secant, and Cotangent

Trig Identities

We will need a couple of identities involving the functions shown above. The major one is: Sin[t]^2 + Cos[t]^2 = 1, that is, sine squared plus cosine squared, equals 1.

Proof:

From the drawing, and definition, Sin[t] = b/c so Sin[t]^2 = b^2/c^2

and, you can see that Cos[t]^2 = a^2/c^2

So adding these I get:

a^2/c^2 + b^2/c^2 = (a^2 + b^2)/c^2

But:

We know that for a right triangle, a^2 + b^2 = c^2, so the ratio becomes ‘1’

Sin[t]^2 + Cos[t]^2 = 1

QED.

All Trig Identities Follow From Euler’s Formula, A Short Detour

This is definitely optional material, unless you want to know how to derive trig identities! If you do want to know more about trig identities I would recommend you find out about Euler’s formula, also called DeMoivre’s law. Looking at this formula requires a short excursion into complex variables but is not too tough if you take it in pieces. Just to whet your appetite, let me state Euler’s formula and then use it to find the double angle formula for Cos[2 t] and Sin[2 t], in terms of Cos[t] and Sin[t].

Euler's Formula

The most important and beautiful formula in all of mathematics is Euler's, stated below. It relates the trig functions to complex numbers in a manner that will allow you to derive all of the trig functions just by applying some simple rules. Below is that formula where 'i' is the square root of -1 and 'e' is a special number that comes up throughout mathematics and is equal to 2.718. I am using the letter 'n' to stand for any number while 't' is an angle (in radian measure). This supremely important formula relates a complex number lying on the unit circle with associated trig functions.

$$e^{i\,n\,t} == \text{Cos}[n\,t] + i\,\text{Sin}[n\,t]$$

FIGURE 5. Euler's Formula (also called DeMoivre's)

Deriving the Trig Identity Cos[2 t] = Cos[t]^2 - Sin[t]^2

When you use complex variables, you get two results from each single derivation. For example, in the derivation below, although we only were looking to see how to express Cos[2t] in terms of Cos[t] and Sin[t], we also got the double angle formula for Sin[2t] with no extra effort.

$e^{i\,t} == \text{Cos}[t] + i\,\text{Sin}[t]$ let n=1 in Euler's formula

$e^{i\,t} * e^{i\,t} = (\text{Cos}[t] + i\,\text{Sin}[t]) * (\text{Cos}[t] + i\,\text{Sin}[t])$

$\text{Cos}[t]^2 + 2\,i\,\text{Cos}[t]\,\text{Sin}[t] - \text{Sin}[t]^2$ multiplying complex numbers

$e^{i\,t} * e^{i\,t} = e^{2\,i\,t}$ use law of exponents e.g. 3^1 * 3^1 = 3^2

$e^{2\,i\,t} == \text{Cos}[2\,t] + i\,\text{Sin}[2\,t]$ let n=2 in Euler's formula

$= \text{Cos}[t]^2 + 2\,i\,\text{Cos}[t]\,\text{Sin}[t] - \text{Sin}[t]^2$

Now equate real parts to real parts and imaginary parts to imaginary parts

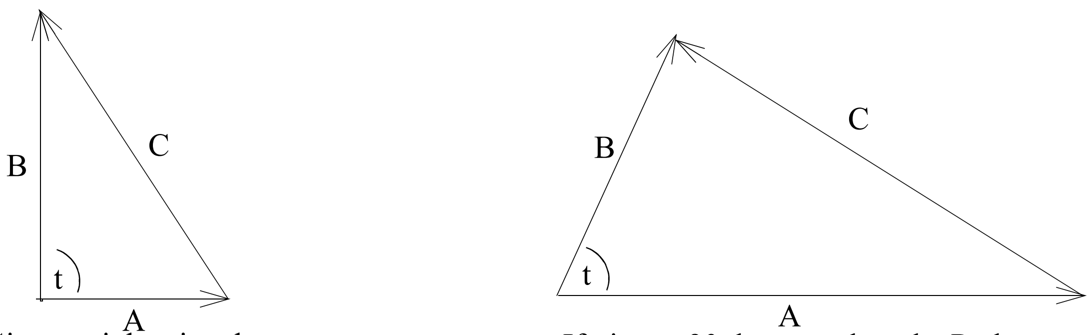
$\text{Cos}[2\,t] = \text{Cos}[t]^2 - \text{Sin}[t]^2$ this is the double angle formula for Cos[2t]

$\text{Sin}[2\,t] = 2\,\text{Cos}[t]\,\text{Sin}[t]$ this is the double angle formula for Sin[2t]

FIGURE 6. Applying Euler's Formula to Derive the Trig Double Angle Formula

Introducing the Law of Cosines

This next rule lets you find the length of a side of a triangle opposite two given sides, even when it is not a right triangle. Look at the diagram below (Figure 7 on page 9:



Given a right triangle (t = 90 degrees) then:
$|C|^2 = |A|^2 + |B|^2$
In words: the squared length of C is the squared length of B plus the squared length of A.

If t is not 90 degrees, then the Pythagorean theorem doesn't (quite) hold. Another identity is used in this case, which has an extra term tacked on:
$|C|^2 = |A|^2 + |B|^2 - 2*|A| * |B| \text{ Cos}[t]$

FIGURE 7. Comparing Side Lengths of a Right Triangle with a General Triangle

Demonstration of the Law of Cosines

The plan here is to describe the length of side C, of a *general triangle*, in terms of the lengths of A, B and the angle 't' between A and B. It turns out that it is easier to first describe how the *square* of C relates to the *squares* of A and B. That is, how |C|^2 relates to |A|^2 and |B|^2. The reason for this is that we will be able to use the Pythagorean theorem as a component of the demonstration.

See if you agree with the steps I will show next:

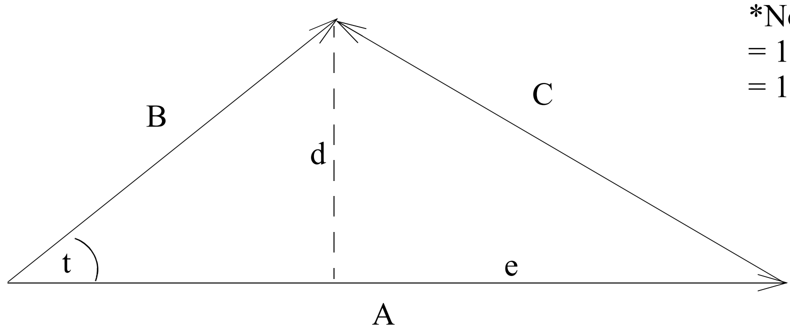
1. drop a perpendicular from the tip of B to A. That is the line whose length is denoted as 'd' and make a right angle with A. Notice that this breaks up the original triangle into two *right* triangles.That is real progress since we have recast the original question by constructing two more questions, but which are simpler. That is, we have broken up a general triangle into two special ones (right triangles) that we know how to work with.
2. the line segment marked 'e' is the length from the tip of A to the base of 'd'.
3. $|C|^2 = d^2 + e^2$ by Pythagoras (and now you know how to prove that!).
4. d = |B| Sin[t] from the diagram, referring to your trig function definition of Sin[t]
5. e = |A| - |B| Cos[t]
6. $|C|^2 = d^2 + e^2 = |B|\text{^}2 * \text{Sin}[t]^2 + |A|^2 + |B|\text{ ^}2* \text{Cos}[t]\text{^}2 - 2*|A| *|B| * \text{Cos}[t]$
7. remember that sine squared plus cosine squared is '1', So, factor out the |B|^2 term from the sine and cosine squares and you will get:
8. |C|^2= d^2 + e^2 = |A|^2 + |B|^2 {Sin[t]^2 + Cos[t]^2} -2*|A| * |B| * Cos[t]
9. |C|^2 = |A|^2 +|B|^2 - 2* |A| * |B|

QED.(quod erat demonstrandom -that which was to be proved)

As an aside, notice that d* A is twice the area of the triangle, so, area =1/2* B Sin[t] * A

A general triangle used to demonstrate the law of cosines

*Note: area of this triangle
= 1/2 d* A
= 1/2 B * Sin[t] * A



$$|C|^2 = d^2 + e^2 = (|B|\text{Sin}[t])^2 + (|A| - |B|\text{Cos}[t])^2$$

$$= |B|^2 \{\text{Sin}[t]^2 + \text{Cos}[t]^2\} + |A|^2 - 2 * |A| * |B| * \text{Cos}[t]$$

$$= |A|^2 + |B|^2 - 2 * |A| * |B| \text{Cos}[t]$$

FIGURE 8. Demo of the Law of Cosines (directed line segments, A, B, and C as *vectors)*

The Dot Product in Terms of the Law of Cosines

I would like to start using the word 'vector' to stand for 'directed line segment'. The use of vectors is everywhere in engineering, physics, statistics, and the social sciences. It's a good idea to start thinking in those terms even with this beginning tutorial. I will use the word vector in the following two sections freely. In all these field, the Dot product also shows up as a way to understand the relation between vectors representing various physical and statistical/mathematical quantities. For example, in engineering, the amount of work done by a force acting through a displacement is given by the product of the component of that force in the direction of the displacement vector. That component can be expressed by the Dot product of the force with the displacement, written as:

F• d = Work

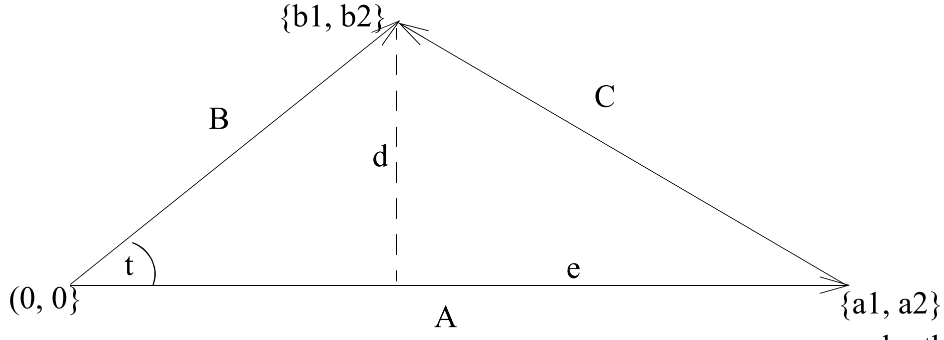
the Dot product of that (constant) force (a vector) with that displacement (a vector) gives the work done by that force (a scalar).

In statistics, the Dot product is used to assess the goodness of a prediction using a predictor. For example If I had a vector representing Income **(I)** and a second representing Education (**E**), how closely are they related? That closeness, called the *correlation coefficient* in statistics, is given by their Dot product **I • E**. Note: technically, the variables have to have their means subtracted off before the Dot product represents their correlation See the tutorial on this site called: *Basic Statistics*)

Now that I have the law of cosines to tell me the length of the directed line segment (vector) 'C' from the above drawing, I can use that to show how the *Dot product* is constructed. Let me now repeat that drawing and add in some coordinates at the tips of vectors A and B, as shown: This means that I have introduced a coordinate system which you can think of as the usual X-Y rectangular coordinate system if you like. The conclusion, derived below, we will come to is that: for two vectors **A** and **B**, the Dot Product looks like:

A • B = |**A**|* |**B**| * Cos[angle between them], ; where |**A**| and |**B**| denote the lengths of **A** and **B**

A general triangle used to demonstrate the Dot Product A • B = |A| *|B| Cos[t]



|C|^2 = |A|^2 + |B|^2 - 2* |A| *|B| * Cos[t] by the law of cosines (derived above)

e^2 + d^2 = Pythagoras theorem, right triangle

(a1- b1)^2 + (a2 - b2)^2 =

a1^2 + a2^2 + b1^2 + b2^2 - 2 (a1 b1 + a2 b2) =

|A|^2 + |B|^2

- 2 (a1 b1 + a2 b2) = - 2* |A| *|B| * Cos[t]

a1 b1 + a2 b2 = |A| *|B| * Cos[t] QED, the Dot Product

FIGURE 9. Coordinate Based Demonstration of the Dot Product, A • B = |A| * |B| * Cos[t]

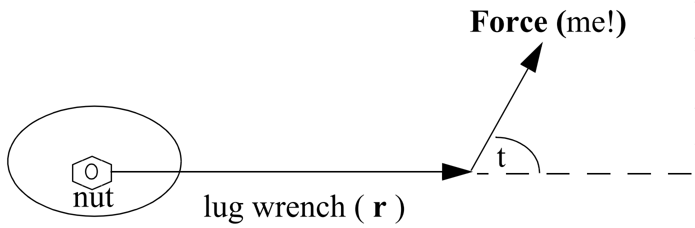
Dot Product as a Test for Perpendicularity

Since we know that the definition of the Dot Product depends on the cosine of the angle between two vectors, and we also know that the cosine of 90 degrees is zero, it follows that if the Dot Product is zero, we have vectors at 90 degrees to each other, that is, they are perpendicular. This is an extremely important criterion for perpendicularity, just take the Dot Produce and see if you get zero!

Cross Product in Terms of Torque and the Area of a Parallelogram

Another operation between vectors is of considerable importance, the Cross product. There is a trig interpretation of this operation which is why I have included it here. Note: I cover this more thoroughly in another tutorial on this site: *Vector Arithmetic & Vector Operations.*

This operations arises most clearly in engineering and physics calculations where the result of the vector operations is another vector which is perpendicular to both of the operand vectors. This needs an example! In engineering there is a quantity called *torque*, which is a measure of how hard it is to get an object rotating. You are familiar with this task when you try to loosen a wheel nut to fix a flat tire. See the diagram below. To remove the nut you use a lug wrench with a long handle. When you apply a force at *right* angles to the wrench handle, the nut turns (hopefully!) Your force, applied at the end of the 'lever' constitutes a *torque*. If you apply your force at right angles to the handle, you will apply more torque. As you adjust your angle of 'push' away from 90 degrees, you diminish your torque and the applied force to the nut. In the extreme case, if you pushed/pulled along the direction of the lug wrench, nothing good will happen. The determining factor here is the *sine* of the angle between your push and the direction of the lug wrench handle. 90 degrees does the best job, which means Sin[90] = 1. Here is a picture that will help!



t is the angle between the lug wrench vector representation and the Force vector. When t = 90 degrees, the maximum torque is applied. When t = 180 or t= 0, nothing good happens!

Cross Product **r x F** = | r | * |F |* Sin[t] **n (**where **n** is a unit vector out of the page, toward you)

The formal definition the cross product of the two vectors wrench, (**r**) and Force, (**F**) is:

torque = **r x F** = |r| * |**F**| * Sin[t] **n**

where ‘t’ is the angle between **r** and **F**. The resultant vector, called *torque*, is a vector perpendicular to both **F** and **r** with magnitude |F| *| r | * Sin[t] and direction **n**. That direction **n** is given by the right hand rule which says: put your fingers along **r**, and rotate your hand into **F**, your thumb then points in the resultant direction, **n**. Think of sliding **r** along the dotted line until its base is at the base of F, and then rotate it into F. The result will be a vector of magnitude |F| *| r | * Sin[t], pointing out of the page.

Area of a Parallelogram of Two Vectors (also known as their Cross Product)

In geometry you have to calculate all kinds of lengths and areas and one of these kinds of areas is called a parallelogram. This is a four sided figure with two parallel sides having the same length. Look at the next figure and notice that the parallelogram is built from two vectors, A and B (you may also note that it is built from two triangles).

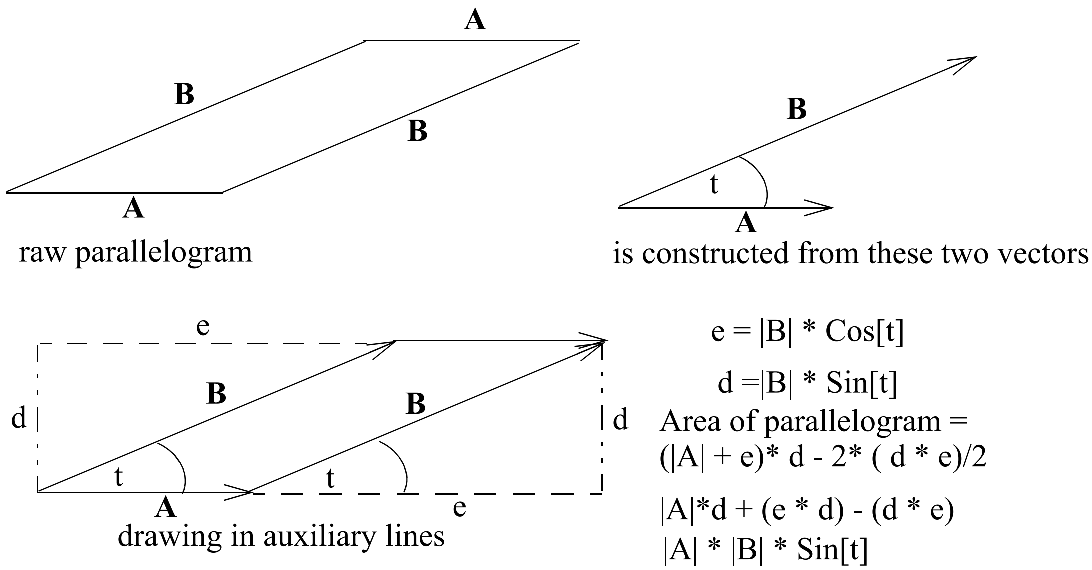


FIGURE 10. Area of a Parallelogram = Cross Product of Two Vectors

From the diagram above you can see how to calculate the area of a parallelogram and by noting an earlier definition of the Cross product of two vectors, you can see that the area of the parallelogram is the magnitude of the Cross product of the vectors used in the construction of the parallelogram.

*As a bonus, notice that the area of the triangle formed by A and B is just half of the parallelogram. So the area of a triangle with included angle is:

Area Triangle = |A| * |B| * Sin[t] /2

Finally, notice that the parallelogram is just the same triangle, joined together twice. We have noticed how to calculate triangle areas earlier within the diagram “Demo of the Law of Cosines (directed line segments, A, B, and C as vectors)” on page 10.

Summary

At this point you have the basics of trig and can move on to the discovery of additional trig identities such as the double angle formulas or half angle formulas. Additionally, I have introduced and proved the law of cosines and used that to show the derivation of the Dot product formula. Cross Products were introduced as well as a brief excursion into analytic geometry using vectors.